HOW UNDERGRADUATE STUDENTS MAKE SENSE OUT OF GRAPHS: THE CASE OF PERIODIC MOTIONS

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This study aims to explore how undergraduate students in mathematics and engineering professions make sense out of graphs representing periodic and repeated but non-periodic motions. In this study, making sense out of graphs means interpreting graphical features and describing a situation that could be represented by them. The data was collected by means of a questionnaire administered to 132 participants. Our findings indicated both students' misconceptions, as every repeated motion is periodical, and their strong willingness to assign practical meaning to mathematical entities.

INTRODUCTION

Any motion that repeats itself identically at regular intervals is called 'periodic motion'. As we observe the periodic motion shown on a graph, we are looking at a function that repeats periodically and sinusoidal functions are of this type (King, 2009). The notion of periodicity is very close to students' experiences since it appears in nature all around us (the annual motion of the earth around the sun, the tides etc.). Moreover, periodicity is a considerable part of the scientific culture of every student in his secondary and post-secondary studies. Particularly, students come to terms with this notion in different school subjects such as mathematics and science (oscillations in physics, periodic functions in trigonometry and calculus) and in post-secondary studies (Fourier series, signal processing etc.). Hence, connecting aspects of periodicity from different school disciplines is important for students' future studies in mathematics, science and engineering. Even though periodicity is central in a variety of disciplines, an extensive search of the literature shows that there is a limited number of studies that focus on its understanding. These studies conclude that most students' concept image of periodicity is based on time-dependent variations (Shama, 1998) while usually they consider any repetition as periodical (Buendia & Cordero, 2005).

The present study is part of a research project that intends to take a close look at pedagogical practices adopted in mathematics and physics classrooms in Greek secondary schools on topics that are related to periodicity. To meet the aims of this inquiry, in the first phase of our project we analyzed Greek physics and mathematics textbooks on selected chapters in the topics of periodic motions and periodic functions respectively (Triantafillou, Spiliotopoulou & Potari, 2013). This analysis has indicated that, when aspects of the notion are introduced, physics adopt a holistic perspective on defining periodic motions, whereas mathematics adopt a point-wise perspective on defining periodic functions ($f(x)$ is a periodic function if there is a positive number p, the period, such that $f(x+p)=f(x)$ for all x in the domain of the function f). We also

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highlighted some common practices in the analysis of the proposed exercises among the subjects, for example, aspects of periodicity are tackled almost exclusively by means of sinusoidal functions and graph related practices were mostly on sketching graphs of particular situations. Furthermore, in physics, functions such as $f(x)=e^{bx}\sin(\omega x)$ that fluctuate in a periodical way on the x-axis, are considered as functions that model periodic motions. This disciplinary understanding of periodicity could encourage incorrect generalizations, such as, any type of repetition is periodical. The aim of the present study is to see if all the above issues will continue to influence undergraduate students' understanding of aspects of periodicity when confronted with the task of making sense of graphical representations of repetitive motions. Our research questions are: (RQ1) How do undergraduate students interpret graphs of periodic motions and do they distinguish them from graphs of repeated but non-periodic motions? (RQ2) What type of examples of motions do they provide that could be represented by graphs of repeated functions? (RQ3) Are there any statistically significant differences between undergraduate students in Mathematics with undergraduate students in Engineering professions, when responding to tasks exploring the above issues?

THEORETICAL FRAMEWORK

We adopt the viewpoint that thinking about physical phenomena could enrich and promote the development of mathematical knowledge (Buendia & Cordero, 2005). Within the school curriculum, graphic competencies are central practices in mathematics and science classrooms (Roth & McGinn, 1997). Different theoretical perspectives have been adopted for analyzing students' making sense of graphs in the mathematical context. From a cognitive perspective, graph sense means "looking at the entire graph (or part of it) and gaining meaning about the relationship between the two variables and, in particular, of their pattern of co-variation" (Leinhardt, Zaslavsky & Stein, 1990, p. 11). Under the embodied cognition perspective, bodily activities are involved in conceptualizing graphical representations as dynamic processes (Nunez, 2007) while from a cultural-semiotic perspective, sensual experiences are important in making sense of motion graphs (Radford, Demers, Guzman & Cerulli, 2004). Moreover, the conceptual movement from graphs to a situation that they represent is termed 'translation' which presupposes the practice of 'making sense out of graphs' (Roth, 2004, p. 77). In the engineering context, translating domain-specific graphs is a central action since graphs mediate collective scientific activities such as communicating and constructing facts (Roth & McGinn, 1997). In the present study, 'making sense out of graphs' means interpreting graphical features and describing a situation that could be represented by them.

METHODOLOGY

The participants were 132 undergraduate students (85 male and 47 female). 19 students were studying Mathematics, 70 were studying Informatics and 43 were studying Electronics. The students were at different stages of the courses (58 were in the second

semester, 45 in the fourth semester and 29 in their sixth or remaining semesters). All mathematics students fall into the last case. At undergraduate level, all students in the above fields encounter aspects of periodicity in their first year Calculus and Fourier analysis courses. Fourier analysis is a prerequisite course for studying signal processing in Informatics and Electronics. Thus, for all the participants, periodicity is considered as an important scientific notion not only for their academic studies, but for their professional life as well.

The tasks: The data was collected by means of a questionnaire administered to the participants at the end of the academic year 2012-13. The questionnaire was completed in one teaching hour during a mathematics course in the case of the engineering students and during a course in mathematics education in the case of students in mathematics. The questionnaire was based on three different practices relating to periodicity (exemplifying; making sense out of graphs; and modelling periodic motions). In the present study we analyze participants' responses to the tasks given in making sense out of graphs that represented repeated motions. In this case, four graphs were given to the students that all represent displacement in meters versus time in seconds. Table 1 shows the four graphs and the resources used.

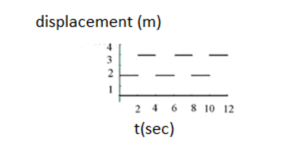
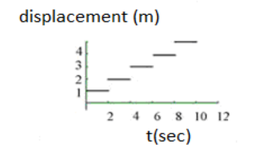
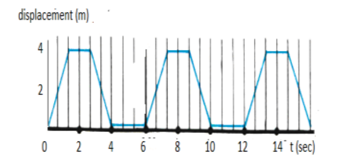
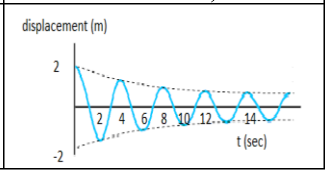
Graph 1 (Buendia & Cordero, 2005)	Graph 2 (Buendia & Cordero, 2005)	Graph 3 (Greek mathematics textbook)	Graph 4 (Greek physics textbook)
			

Table 1: The Graphs

Graph 1 and Graph 3 represent periodic motions while Graph 2 and Graph 4 represent non-periodic motions. Moreover, Graph 1 and Graph 2 represent non-continuous motions. Two tasks were given to the students referring to each graph separately. Task 1: *Does this graph represent a periodic motion? Justify your answer.* In this task, students are asked to focus on how the repetition is accomplished in order to distinguish the periodic from the non-periodic motions, as well as justifying their response. Task 2: *Provide an example that could be described by this particular graph*. In this task the students were asked to assign to each graph a motion that could be represented by it.

Data analysis: Qualitative content analysis has been employed for the analysis of students' responses in both tasks (Mayring, 2000). All the categories emerged from our continuous interrogation of the data. We separated students' responses in distinguishing periodical from non-periodical motions (Task 1a); justifying their responses (Task 1b); analyzing the situations that were created by the students in respect of the salient features of the graphs they took into consideration (Task 2a); and categorizing the type of examples used by the students (Task 2b). Subsequently, we reported the frequency and the valid percentage of students' responses on the categories that emerged from both tasks for the four graphs. Finally, in order to

compare mathematics and engineering students' responses on the emerging categories we used the goodness of fit test at 0.05 level of significance.

FINDINGS

We present the results of our analysis of the categories that emerged from the students' responses on the two tasks in the case of each graph and we provide some characteristic examples. Finally, we present the cases of statistical significant differences between the mathematics and the engineering students. In Tables 2, 3, 4 and 5, we present the percentages of students' responses in the categories that emerged from each task across the four graphs and the number of students that responded to the particular item.

Task 1a: Does this graph represent a periodic motion? Two categories emerged from the analysis of this task: the graph represents a *non-periodic motion*; and the graph represents a *periodic motion*.

Categories	Graph 1	Graph 2	Graph 3	Graph 4
	113 Participants	107 Participants	109 Participants	114 Participants
Non-Periodic	23.89	74.77	7.34	32.46
Periodic	76.11	24.30	92.66	67.55

Table 2: % of students' responses across categories and across graphs on Task 1a

Almost three out of four students identified periodicity in Graph 1 and non-periodicity in Graph 2 while this percentage increases in the case of Graph 3 since more than nine out of ten students identified it as a periodic graph. Graph 4, which represents a repeated but a non-periodic motion, seemed to confuse students a lot since almost seven out of ten considered it to be a periodic graph. Comparing mathematics and engineering students' responses the goodness of fit test showed statistical significant differences between them only in the case of Graph 4 (Pearson Chi square value 9.527 and p= 0.009) since more than half of them (11 out of 17) considered it a case of non-periodic motion. However, this result does not change our hypothesis that students have difficulties distinguishing periodic from non-periodic motions.

Task 1b: Justify your answer (in task 1a). The following categories emerged from the analysis of students' responses: *Referring to general patterns of repetition, relating variations in x-y axis, focusing on continuity issues, using the formal definition of periodic functions,* and *reasoning on a specific situation*.

Categories	Graph 1	Graph 2	Graph 3	Graph 4
	59 Participants	46 Participants	58 Participants	63 Participants
Referring to general patterns of repetition	23.73	23.91	32.76	15.87
Relating variations in x-y axis	37.29	26.09	41.38	61.90
Focusing on continuity issues	13.56	15.22	1.72	1.59
Using a formal definition	0.00	2.17	1.72	0.00
Reasoning on a specific situation	25.42	32.61	22.41	20.63

Table 3: % of students' responses in the categories and across graphs on Task1b

In both cases of periodic and non-periodic graphs, students preferred to justify their answers by relating patterns of repetitions between the x-axis and y-axis rather than referring to general patterns of repetition. It is interesting that the same type of the above justifications were used for conflicting answers for the same graph. Two characteristic responses of the type of *General pattern* in the case of Graph1 are the following: "It is periodic because we have a repetition" (st91_elec) and, "It is not periodic since there is not any harmony" (st59_elec). In the following examples we could identify inconsistencies in the students' responses in the case of Graph 4 when *relating patterns of variations in the x-axis and the y-axis*: "It is periodic but we can see that as the time passes it dwindles and we are led to a standstill" (st101_elec); or "It is a periodic motion that decreases (its amplitude diminishes) all the time" (st68_elec). The contradictions in students' responses were not realized by them. *Focusing on the continuity issue* is used as a warrant to take the stance that Graphs 1 and 2 are both non-periodic. For example, st19_math notices: "I do not know if this graph preserves a periodic behaviour because in its second position it has different values from left and right". Only st6_math reasons by *using the definition of periodic functions* in order to accept that Graph 3 is periodic and Graph 2 is non-periodic. For example, Graph 3 "is periodic with period T=6 seconds since f(x+T)=f(x) for every x in the interval [0,14]. In the case of Graph 4, the same student changes his argument as follows: "It is periodic since any sinusoidal function is periodic". The last category is *reasoning on a specific situation*. These situations, in most cases, were the examples they provided in Task 2. This type of situated justification was common in students' responses in all graphs and ranged from 20% to 30%. Some characteristic examples are: (Graph 1) "the body of the graph diverges from the starting point of motion and then always returns within 4 seconds, therefore the graph is periodic (st99_elec); (Graph 2) "the graph shows a person who, as time passes, only draws away from a point 'a,' therefore non-periodic" (st107_inf); and (Graph 4) "it is periodic because it represents the motion of the swing" (st129_inf). This indicates students' need to set up a background for their justifications.

Finally, comparing mathematics and engineering students' responses on the emerging categories the goodness of fit test showed statistical significant differences between them only in the case of Graph 2 (Pearson Chi square value 11.138 and p= 0.025) since they rarely used situated type justifications.

Task 2: Provide an example of motion that could be described by each graph.

Task 2a: The following categories emerged from the analysis of the salient features that were taken into consideration when the students were asked to provide examples that could be represented by each graph: *Enriched repeated motion* when students considering the repeated behaviour and other characteristics emerging from the graphs (periodicity, piece-wise continuity, and the relation between the variables), *Only repeated motions* when students took into consideration only the repeated behaviour, Non-repeated motions when there was no-indication of a repeating motion in students' responses, and *no-motion* when the example was not representing a motion at all.

Categories	Graph 1	Graph 2	Graph 3	Graph 4
	84 Participants	83 Participants	85 Participants	102 Participants
Enriched repeated motions	9.52	21.69	7.06	6.86
Only repeated motions	55.95	31.32	67.06	79.41
Non-repeated motions	15.48	35.55	12.94	0.98
Non-motions	19.05	8.43	12.94	12.75

Table 4: % of students' responses in the categories and across graphs on Task 2a

Creating a motion example of a piece-wise continuous function is very difficult but a few students managed to provide examples that could satisfy all the graphical features in these graphs. In this case, students used their kinesthetic experiences of 'jumping' or 'climbing stairs' in order to respond to this task. Some typical examples of *enriched repeated motions* in the case of Graph 1: "ascending and descending jumps between uneven steps (st1_math)"; in the case of Graph 2 is: "someone who is climbing stairs" (st57_inf). Noticing the resemblance of Graph 2 with stairs and visualize the motions helped almost 22% of the participants to provide enriched examples. However, Graphs 3 and 4 were more complicated since the students had to take into consideration the type of co-variance of the two variables in order to provide enriched examples. Particularly, Graph 3 refers to an object's motion that moves with constant speed in different directions and makes a few seconds stops. A significant example for Graph 3 is: "someone who is using a piece of gym equipment which is going to and fro with constant speed and stops for a few seconds" (st59_elec). Although many students used the swing example for Graph 4, a typical example in their physics classes, only a few managed to specify what the x-axis and the y-axis represent in this graph. This is the reason we have the least percentage of enriched cases.

The number of students who provided *examples of repeated motions* but did not consider other graphical features was high (more than one out of two students) for all graphs besides Graph 2. Two characteristic ideas were met in their answers and the corresponding examples for the graphs follow: (a) discontinuity was not taken into consideration (Graph 1) "it represents an elevator that is trapped going up and down between the second and fourth floor" (st3_math); (b) not specifying the x-y co-variance (Graph 3) "two people who are throwing a ball to each other" (st50_inf). The amount of students who provided examples of *non- repeated motions* was higher in the case of Graph 2, as for example: "a dog that goes hunting and increases its speed" (st71_elec). Some students provided *examples that do not represent motions* at all. These examples are mostly taken from their academic signal processing courses. The goodness of fit test did not indicate any statistical significant differences in mathematics students' responses. Finally, students' high participation in this task indicates their willingness to assign meaning to abstract mathematical entities.

Task 2b: Kinesthetic students' experiences seem to play a significant role in providing examples of repeated motions. So, we further analyzed the type of kinesthetic experiences they refer to and the categories emerged were: *bodily actions* when a

human agent performs the motion (an athlete running or a frog jumping), *physical tools motions* (a car is accelerating or a swing is oscillating), and *vibrations of natural objects* (a sea wave or a sound wave

Categories	Graph 1	Graph 2	Graph 3	Graph 4
	70 Participants	81 Participants	74 Participants	90 Participants
Bodily actions	24.29	32.50	13.51	7.78
Physical tools' motions	72.14	63.80	81.09	64.44
Vibrations of natural objects	3.57	3.70	5.40	27.78

Table 5: % of students' responses in the categories and across graphs on Task 2b

Physical tools' motions provided the context used by most students to translate the graphs to situations. The highest percentage of *bodily actions* examples was in the case of Graph 2 (32.5%). The highest percentage of *physical tools' motions* (81%) was in the case of Graph 3. We interpret this result that most students consider that human actions are very difficult to model this type of motion graphs so they have changed the context of their example from bodily actions to physical tools' motions. More than one out of four students used examples of *vibrating natural objects* (e.g. sea waves) in describing the case of Graph 4. The graphical image resemblance with traveling sinusoidal waves was the reason to use them as the context of their examples. We note that waves are functions of two variables, the displacement x and the time t (King, 2013). Finally, the goodness of fit test showed high statistical significant differences in mathematics students' responses in the case of Graph 2 (Pearson Chi square value 13.547 and p= 0.001) since math students exclusively used examples of bodily motions when describing this graph.

CONCLUDING REMARKS

This study aims to explore how undergraduate students in mathematics and engineering professions make sense out of graphs representing periodic and repeated but non-periodic motions. Our findings indicate that conceptions such as "every repeated motion is periodic" or "any sinusoidal graph, even with decreasing amplitude, represents a periodic motion" dominate students' understanding. Even mathematics students seem not to realize the above contradictions in their responses.

However, students' strong willingness to assign meaning to mathematical entities is proved both by their high participation in providing situations that could fit onto motion graphs and by the fact that they use these situations as warrants for their justifications. In this case, the role of students' kinesthetic experiences proved central both when they provided enriched examples of motions represented by the particular graphs and when they take the stance to change the context of the examples according to their perception of the graphical features represented. These findings show the embodied nature of mathematical thinking and the genetic relationship between the sensual and the conceptual in knowledge formation (Nunez, 2007; Radford et al., 2004). Translating graphs into describing situations (Roth, 2004) seems to be an activity that attracts undergraduate students' attention. Maybe such activities help

students to cope with the contradictions that arise between their divergent conceptions on periodicity. The formal mathematical tools, as the definition of periodic functions, seem to be not enough to change such perceptions even in the case of students who study mathematics.

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References

Buendia G., & Cordero F. (2005). Prediction and the periodical aspect as generators of knowledge in a social practice framework. *Educational Studies in Mathematics*, *58*(3), 299-333.

King, G. C. (2013). *Vibrations and waves*. Hoboken, NJ: Wiley publishers.

Leinhardt, G, Zaslavsky, O, & Stein M. (1990). Functions, graphs and graphing: Tasks, learning and teaching. *Review of Educational Research*, *60*(1), 37-42.

Mayring, P. (2000). Qualitative content analysis. *Forum Qualitative Sozialforschung – Forum: Qualitative Social Research*, *1*(2).
Retrieved from http://nbn-resolving.de/urn:nbn:de:0114-fqs0002204

Nunez, R. (2007). The cognitive science of mathematics: Why is it relevant for mathematics education? In R. A. Lesh, E. Hamilton, & J. Kaput (Eds.), *Foundations for the future in mathematics education* (pp. 127-154). London: Lawrence Erlbaum Associates.

Radford, L., Cerulli, M., Demers, S., & Guzmán, J. (2004). The sensual and the conceptual: Artefact mediated kinesthetic actions and semiotic activity. In M. J. Høines & A. B. Fuglestad (Eds.), *Proc. 28th Conf. of the Int. Group for the Psychology of Mathematics Education* (Vol. 4, pp. 73-80). Norway: PME.

Roth, W. M. (2004). What is the meaning of "meaning"? A case study from graphing. *Journal of Mathematical Behavior*, *23*, 75-92

Roth, W., & McGinn, M.K. (1997). Graphing: Cognitive ability or practice? *Science Education*, *81*, 91-106.

Shama, G. (1998). Understanding periodicity as a process with a gestalt structure. *Educational Studies in Mathematics*, *35*, 255-281.

Triantafillou, C., Spiliotopoulou, V., & Potari, D. (2013). Periodicity in textbooks: Reasoning and visual representations. In A. M. Lindmeier & A. Heinze (Eds.), *Proc. 37th Conf. of the Int. Group for the Psychology of Mathematics Education* (Vol. 4, pp. 297-304). Kiel, Germany: PME.